1. The discrete random variable x has probability distribution given by

x	–1	0	1	2	3
$P(X = x)$	$\frac{1}{5}$	a	$\frac{1}{10}$	a	$\frac{1}{5}$

where a is a constant.

(a) Find the value of a.

(2)

(b) Write down $E(X)$.

(1)

(c) Find $Var(X)$.

(3)

The random variable $Y = 6 - 2X$

(d) Find $Var(Y)$.

(2)

(e) Calculate $P(X \geq Y)$.

(3)
(Total 11 marks)

2. The probability function of a discrete random variable x is given by

$$p(x) = kx^2 \qquad x = 1, 2, 3$$

where k is a positive constant.

(a) Show that $k = \frac{1}{14}$

(2)

Find

(b) $P(X \geq 2)$

(2)

(c) $E(X)$

(2)

(d) $Var(1 - X)$

(4)

(Total 10 marks)

3. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} a(3-x) & x = 0,1,2 \\ b & x = 3 \end{cases}$$

(a) Find $P(X = 2)$ and complete the table below.

x	0	1	2	3
$P(X = x)$	$3a$	$2a$		b

(1)

Given that $E(X) = 1.6$

(b) Find the value of a and the value of b.

(5)

Find

(c) $P(0.5 < X < 3)$,

(2)

(d) $E(3X - 2)$.

(2)

(e) Show that the Var(X) = 1.64

(3)

(f) Calculate Var($3X$ – 2).

(2)
(Total 15 marks)

4. When Rohit plays a game, the number of points he receives is given by the discrete random variable X with the following probability distribution.

x	0	1	2	3
P($X = x$)	0.4	0.3	0.2	0.1

(a) Find E(X).

(2)

(b) Find F(1.5).

(2)

(c) Show that Var(X) = 1

(4)

(d) Find Var(5 – 3X).

(2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10.
After 3 games he has a total of 6 points.
You may assume that games are independent.

(e) Find the probability that Rohit wins the prize.

(6)
(Total 16 marks)

5. The random variable X has probability distribution given in the table below.

X	–1	0	1	2	3
$P(X = x)$	p	q	0.2	0.15	0.15

Given that $E(X) = 0.55$, find

(a) the value of p and the value of q,
(5)

(b) $\text{Var}(X)$,
(4)

(c) $E(2X - 4)$.
(2)
(Total 11 marks)

6. The discrete random variable X can take only the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$F(x) = \frac{(x+k)^2}{25} \text{ for } x = 2,3,4$$

where k is a positive integer.

(a) Find k.
(2)

(b) Find the probability distribution of X.
(3)
(Total 5 marks)

7. The random variable X has probability distribution

x	1	3	5	7	9
$P(X = x)$	0.2	p	0.2	q	0.15

(a) Given that $E(X) = 4.5$, write down two equations involving p and q.
(3)

Find

(b) the value of p and the value of q,

(3)

(c) $P(4 < X \leq 7)$.

(2)

Given that $E(X^2) = 27.4$, find

(d) $\text{Var}(X)$,

(2)

(e) $E(19 - 4X)$,

(1)

(f) $\text{Var}(19 - 4X)$.

(2)
(Total 13 marks)

8. The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{5}, \qquad x = 1, 2, 3, 4, 5.$$

(a) Write down the value of $E(X)$ and show that $\text{Var}(X) = 2$.

(3)

Find

(b) $E(3X - 2)$,

(2)

(c) $\text{Var}(4 - 3X)$.

(2)
(Total 7 marks)

9. The random variable X has probability distribution

x	1	2	3	4	5
$\mathrm{P}(X = x)$	0.10	p	0.20	q	0.30

(a) Given that $\mathrm{E}(X) = 3.5$, write down two equations involving p and q.

(3)

Find

(b) the value of p and the value of q,

(3)

(c) $\mathrm{Var}(X)$,

(4)

(d) $\mathrm{Var}(3 - 2X)$.

(2)
(Total 12 marks)

10. The random variable X has probability function

$$\mathrm{P}(X = x) = \begin{cases} kx, & x = 1, 2, 3, \\ k(x+1), & x = 4, 5, \end{cases}$$

where k is a constant.

(a) Find the value of k.

(2)

(b) Find the exact value of $\mathrm{E}(X)$.

(2)

(c) Show that, to 3 significant figures, $\mathrm{Var}(X) = 1.47$.

(4)

(d) Find, to 1 decimal place, Var(4 – 3X).

(2)
(Total 10 marks)

11. The random variable X has probability function

$$P(X = x) = kx, \qquad x = 1, 2, ..., 5.$$

(a) Show that $k = \frac{1}{15}$.

(2)

Find

(b) P(X < 4),

(2)

(c) E(X),

(2)

(d) E(3X – 4).

(2)
(Total 8 marks)

12. A discrete random variable X has a probability function as shown in the table below, where a and b are constants.

x	0	1	2	3
P(X = x)	0.2	0.3	b	a

Given that E(X) = 1.7,

(a) find the value of a and the value of b.

(5)

Find

(b) $P(0 < X < 1.5)$,

(1)

(c) $E(2X - 3)$.

(2)

(d) Show that $Var(X) = 1.41$.

(3)

(e) Evaluate $Var(2X - 3)$.

(2)
(Total 13 marks)

13. A discrete random variable X has the probability function shown in the table below.

x	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{12}$	$\frac{1}{12}$

Find

(a) $P(1 < X \leq 3)$,

(2)

(b) $F(2.6)$,

(1)

(c) $E(X)$,

(2)

(d) $E(2X - 3)$,

(2)

(e) Var(X)

(3)
(Total 10 marks)

14. A fairground game involves trying to hit a moving target with a gunshot. A round consists of up to 3 shots. Ten points are scored if a player hits the target, but the round is over if the player misses. Linda has a constant probability of 0.6 of hitting the target and shots are independent of one another.

(a) Find the probability that Linda scores 30 points in a round.

(2)

The random variable X is the number of points Linda scores in a round.

(b) Find the probability distribution of X.

(5)

(c) Find the mean and the standard deviation of X.

(5)

A game consists of 2 rounds.

(d) Find the probability that Linda scores more points in round 2 than in round 1.

(6)
(Total 18 marks)

15. The random variable X has the discrete uniform distribution

$$P(X = x) = \frac{1}{n}, \qquad x = 1, 2, ..., n.$$

Given that E(X) = 5,

(a) show that n = 9.

(3)

Find

(b) $P(X < 7)$,

(2)

(c) Var (X).

(4)
(Total 9 marks)

16. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(x^2 - 9), & x = 4, 5, 6 \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = \dfrac{1}{50}$.

(3)

(b) Find $E(X)$ and $\text{Var}(X)$.

(6)

(c) Find $\text{Var}(2X - 3)$.

(2)
(Total 11 marks)

17. The random variable X represents the number on the uppermost face when a fair die is thrown.

(a) Write down the name of the probability distribution of X.

(1)

(b) Calculate the mean and the variance of X.

(3)

Three fair dice are thrown and the numbers on the uppermost faces are recorded.

(c) Find the probability that all three numbers are 6.

(2)

(d) Write down all the different ways of scoring a total of 16 when the three numbers are added together.

(4)

(e) Find the probability of scoring a total of 16.

(2)
(Total 12 marks)

18. The discrete random variable X has probability function

$$P(X = x) = \begin{cases} k(2-x), & x = 0, 1, 2, \\ k(x-2), & x = 3, \\ 0, & \text{otherwise,} \end{cases}$$

where k is a positive constant.

(a) Show that $k = 0.25$.

(2)

(b) Find E(X) and show that E(X^2) = 2.5.

(4)

(c) Find Var($3X - 2$).

(3)

Two independent observations X_1 and X_2 are made of X.

(d) Show that P($X_1 + X_2 = 5$) = 0.

(1)

(e) Find the complete probability function for $X_1 + X_2$.

(3)

(f) Find P($1.3 \leq X_1 + X_2 \leq 3.2$).

(3)
(Total 16 marks)

19. A customer wishes to withdraw money from a cash machine. To do this it is necessary to type a PIN number into the machine. The customer is unsure of this number. If the wrong number is typed in, the customer can try again up to a maximum of four attempts in total. Attempts to type in the correct number are independent and the probability of success at each attempt is 0.6.

(a) Show that the probability that the customer types in the correct number at the third attempt is 0.096.

(2)

The random variable A represents the number of attempts made to type in the correct PIN number, regardless of whether or not the attempt is successful.

(b) Find the probability distribution of A.

(2)

(c) Calculate the probability that the customer types in the correct number in four or fewer attempts.

(2)

(d) Calculate $\mathrm{E}(A)$ and $\mathrm{Var}(A)$.

(6)

(e) Find $\mathrm{F}(1 + \mathrm{E}(A))$.

(2)
(Total 14 marks)

1. (a)	$2a + \frac{2}{5} + \frac{1}{10} = 1$	(or equivalent)	M1	
	$\underline{a = \frac{1}{4}\ \textbf{or 0.25}}$		A1	2

Note

M1for a clear attempt to use $\sum$ P(X = x) = 1

Correct answer only 2/2.
NB Division by 5 in parts (b), (c) and (d) seen scores 0.
Do not apply ISW.

(b)	E(X) = **<u>1</u>**		B1	1

Note

B1 for 1

(c)	E(X^2) $= 1\times\frac{1}{5} + 1\times\frac{1}{10} + 4\times\frac{1}{4} + 9\times\frac{1}{5}$	(= 3.1)	M1	
	Var(X) = $3.1 - 1^2$,	$\underline{= \textbf{2.1 or } \frac{21}{10}\ \textbf{oe}}$	M1 A1	3

Note

1st M1 for attempting $\sum x^2 \mathrm{P}(X = x)$ at least two terms correct.
Can follow through.
2nd M1 for attempting E(X^2) – [E(X)]^2 or allow subtracting 1 from their attempt at E(X^2) provided no incorrect formula seen.
Correct answer only 3/3.

(d)	Var(Y) = $(-2)^2$ Var(X),	= $\underline{\textbf{8.4 or } \frac{42}{5}\ \textbf{oe}}$	M1 A1	2

Note

M1 for $(-2)^2$ Var(X) or 4Var(X)
Condone missing brackets provided final answer correct for their Var(X).
Correct answer only 2/2.

(e)	$X \geq Y$ when X = 3 or 2, so probability = "$\frac{1}{4}$"+ $\frac{1}{5}$		M1 A1ft	
		= $\underline{\frac{9}{20}\ \textbf{oe}}$	A1	3

Note

Allow M1 for distribution of $Y = 6 - 2X$ and correct attempt at E(Y^2) – [E(Y)]^2

M1 for identifying X = 2, 3
1st A1ft for attempting to find their P(X = 2) + P(X = 3)
2nd A1 for $\frac{9}{20}$ or 0.45

[11]

2. (a)	$k + 4k + 9k = 1$		M1	
	$14k = 1$			
	$k = \frac{1}{14}$ * * given * *	cso	A1	2

Note

M1 for clear attempt to use $\sum p(x) = 1$, full expression needed and the "1" must be clearly seen. This may be seen in a table.

A1cso for no incorrect working seen. The sum and "= 1" must be explicitly seen somewhere.

A verification approach to (a) must show addition for M1 and have a suitable comment e.g. "therefore $k = \frac{1}{14}$" for A1 cso

(b)	$P(X \geq 2) = 1 - P(X = 1)$ or $P(X = 2) + P(X = 3)$		M1	
	$= 1 - k = \frac{13}{14}$ or 0.92857...	**awrt 0.929**	A1	2

Note

M1 for 1- P(X \≤ 1) or P(X = 2) + P(X = 3)

A1 for awrt 0.929. Answer only scores 2/2

(c)	$E(X) = 1\times k + 2\times k \times 4 + 3\times k \times 9$ or $36k$		M1	
	$= \frac{36}{14} = \frac{18}{7}$ or $2\frac{4}{7}$	(or exact equivalent)	A1	2

Note

M1 for a full expression for E(X) with at least two terms correct.

NB If there is evidence of division (usually by 3) then score M0

A1 for any exact equivalent – answer only scores 2/2

(d)	$\text{Var}(X) = 1\times k + 4 \times k \times 4 + 9 \times k \times 9, -\left(\frac{18}{7}\right)^2$	M1 M1	
	$\text{Var}(1-X) \quad = \text{Var}(X)$	M1	
	$\frac{19}{49}$ or 0.387755... **awrt 0.388**	A1	4

Note

1st M1 for clear attempt at $E(X^2)$, need at least 2 terms correct in $1\times k + 4\times 4k + 9 \times 9k$ or $E(X^2) = 7$

2nd M1 for their $E(X^2)$ – (their μ)^2

3rd M1 for clearly stating that $\text{Var}(1 - X) = \text{Var}(X)$, wherever seen

A1 accept awrt 0.388. All 3 M marks are required.

Allow 4/4 for correct answer only but must be for $\text{Var}(1 - X)$.

[10]

3. (a)

0	1	2	3
$3a$	$2a$	a	b

B1 1

Note

Condone a clearly stated in text but not put in table.

(b)	$3a + 2a + a + b = 1$	or equivalent, using Sum of probabilities =1	M1	
	$2a + 2a + 3b = 1.6$	or equivalent, using $E(X)$ =1.6	M1	
	$14a = 1.4$	Attempt to solve	M1dep	
	$a = 0.1$	cao	B1	
	$b = 0.4$	cao	B1	5

Note

Must be attempting to solve 2 different equations so third M dependent upon first two Ms being awarded.

Correct answers seen with no working B1B1 only, 2/5

Correctly verified values can be awarded M1 for correctly verifying sum of probabilities

=1, M1 for using E(X)=1.6 M0 as no attempt to solve and B1B1 if answers correct.

(c)	$P(0.5 < x < 3) = P(1) + P(2)$	3a or their $2a$+their a	M1	
	$= 0.2 + 0.1$			
	$= 0.3$	Require 0<3a<1 to award follow through	A1 ft	2

(d)	$E(3X - 2) = 3E(X) - 2$		M1	
	$= 3 \times 1.6 - 2$			
	$= 2.8$	cao	A1	2

Note

2.8 only award M1A1

(e)	$E(X^2) = 1\times 0.2 + 4 \times 0.1 + 9 \times 0.4$ (= 4.2)		M1	
	Var (X) = "4.2" – 1.6^2		M1	
	$= 1.64$ * * given answer * *	cso	A1	3

Note

Award first M for at least two non-zero terms correct. Allow first M for correct expression with a and b e.g. $E(X^2) = 6a+9b$

Given answer so award final A1 for correct solution.

(f)	$Var(3X - 2) = 9\ Var(X)$		M1	
	$= 14.76$	awrt 14.8	A1	2

Note

14.76 only award M1A1

[15]

4. (a)	$E(X) = 0 \times 0.4 + 1 \times 0.3 + ... + 3 \times 0.1, = 1$	M1, A1	2

Note

M1 for at least 3 terms seen. Correct answer only scores M1A1. Dividing by $k(\neq 1)$ is M0.

(b)	$F(1.5) = [P(X \leq 1.5) =]\ P(X \leq 1), = 0.4 + 0.3 = 0.7$	M1, A1	2

Note

M1 for $F(1.5) = P(X \leq 1)$.[**Beware**: $2 \times 0.2 + 3 \times 0.1 = 0.7$ but scores M0A0]

(c) $E(X^2) = 0^2 \times 0.4 + 1^2 \times 0.3 + ... + 3^2 \times 0.1, = 2$ M1, A1

$Var(X) = 2 - 1^2, = 1$ (*) M1, A1cso 4

Note

1st M1 for at least 2 non-zero terms seen. $E(X^2) = 2$ alone is M0.
Condone calling $E(X^2) = Var(X)$.

ALT

1st A1 is for an answer of 2 or a fully correct expression.
2nd M1 for $-\mu^2$, condone $2 - 1$, unless clearly $2 - \mu$ Allow $2 - \mu^2$ with $\mu = 1$ even if $E(X) \neq 1$
2nd A1 for a fully correct solution with no incorrect working seen, **both** Ms required.
$\Sigma (x - \mu)^2 \times P(X = x)$
1st M1 for an attempt at a full list of $(x - \mu)^2$ values and probabilities.
1st A1 if all correct
2nd M1 for at least 2 non-zero terms of $(x - \mu)^2 \times P(X = x)$ seen. 2nd A1 for $0.4 + 0.2 + 0.4 = 1$

(d) $Var(5 - 3X) = (-3)^2 Var(X), = 9$ M1, A1 2

Note

M1 for use of the correct formula. $-3^2 Var(X)$ is M0 unless the final answer is >0.

(e)

Total	Cases	Probability	
4	$(X = 3)\cap(X = 1)$	$0.1 \times 0.3 = 0.03$	
	$(X = 1)\cap(X = 3)$	$0.3 \times 0.1 = 0.03$	
	$(X = 2)\cap(X = 2)$	$0.2 \times 0.2 = 0.04$	B1B1B1
5	$(X = 3)\cap(X = 2)$	$0.1 \times 0.2 = 0.02$	
	$(X = 2)\cap(X = 3)$	$0.2 \times 0.1 = 0.02$	M1
6	$(X = 3)\cap(X = 3)$	$0.1 \times 0.1 = 0.01$	A1

Total probability = $0.03 + 0.03 + 0.04 + 0.02 + 0.02 + 0.01 = 0.15$ A1 6

Note

Can follow through their $Var(X)$ for M1

ALT

1st B1 for all cases listed for a total of 4 or 5 or 6 . e.g. (2,2) counted twice for a total of 4 is B0
2nd B1 for all cases listed for 2 totals }
3rd B1 for a complete list of all 6 cases } These may be highlighted in a table
Using Cumulative probabilities
1st B1 for one or more cumulative probabilities used e.g.2 then 2

or more or 3 then 1 or more

2nd B1 for both cumulative probabilities used. 3rd B1 for a complete list 1, 3; 2, ≥2; 3, ≥1

M1 for one correct pair of correct probabilities multiplied

1st A1 for all 6 correct probabilities listed (0.03, 0.03, 0.04, 0.02, 0.02, 0.01) needn't be added.

2nd A1 for 0.15 or exact equivalent only as the final answer.

[16]

5. (a) $-1 \times p + 1 \times 0.2 + 2 \times 0.15 + 3 \times 0.15 = 0.55$ M1dM1

$p = 0.4$ A1

$p + q + 0.2 + 0.15 + 0.15 = 1$ M1

$q = 0.1$ A1 5

M1 for at least 2 correct terms on LHS
Division by constant e.g. 5 then M0
dM1 dependent on first M1 for equate to 0.55 and attempt to solve.
Award M1M1A1 for p=0.4 with no working
M1 for adding probabilities and equating to 1.
All terms or equivalent required e.g. $p + q = 0.5$
Award M1A1 for $q = 0.1$ with no working

(b) $\text{Var}(X) = (-1)^2 \times p + 1^2 \times 0.2 + 2^2 \times 0.15 + 3^2 \times 0.15, -0.55^2$ M1A1,M1

$= 2.55 - 0.3025 = 2.2475$ awrt 2.25 A1 4

M1 attempting $E(X^2)$ with at least 2 correct terms
A1 for fully correct expression or 2.55
Division by constant at any point e.g. 5 then M0
M1 for subtracting their mean squared
A1 for awrt 2.25
Award awrt 2.25 only with no working then 4 marks

(c) $E(2X - 4) = 2E(X) - 4$ M1

$= -2.9$ A1 2

M1 for 2x(their mean) –4
Award 2 marks for –2.9 with no working

[11]

6. (a) $F(4) = 1$

$(4 + k)^2 = 25$ M1

$k = 1$ as $k > 0$ A1 2

M1 for use of F(4) = 1 only If F(2) = 1 and / or F(3) = 1 seen then M0.
F(2) + F(3) + F(4) = 1 M0
A1 for $k = 1$ and ignore $k = -9$

(b)

x	2	3	4

$P(X = x)$	$\frac{9}{25}$	$\frac{7}{25}$	$\frac{9}{25}$

B1ftB1B1 3

B1ft follow through their k for $P(X = 2)$ either exact or 3sf between 0 and 1 inclusive.
B1 correct answer only or exact equivalent
B1 correct answer only or exact equivalent

[5]

7. (a) $p + q = 0.45$ B1
$\Sigma x P(X = x) = 4.5$ M1
$3p + 7q = 1.95$ A1 3

$0.55 + p + q = 1$ award B1. Not seen award B0.
$0.2 + 3p + 1 + 7q + 1.35 = 4.5$ or equivalent award M1A1
$3p + 7q + k = 4.5$ award M1.

(b) Attempt to solve equations in (a) M1
$q = 0.15$ A1
$p = 0.30$ A1 3

Attempt to solve must involve 2 linear equations in 2 unkowns
Correct answers only for accuracy.
Correct answers with no working award 3/3

(c) $P(4 < X < 7) = P(5) + P(7)$ M1
$= 0.2 + q = 0.35$ A1ft 2

Follow through accuracy mark for their q, $0 < q < 0.8$

(d) $\text{Var}(X) = E(X^2) - [E(X)]^2 = 27.4 - 4.5^2$ M1
$= 7.15$ A1 2

Attempt to substitute <u>given</u> values <u>only</u> into correct formula for M1.
7.15 only for A1
7.15 seen award 2/2

(e) $E(19 - 4X) = 19 - 4 \times 4.5 = 1$ B1 1

(f) $\text{Var}(19 - 4X) = 16\text{Var}(X)$ M1
$= 16 \times 7.15 = 114.4$ A1 2

Accept 'invisible brackets' i.e. $-4^2\ \text{Var}(X)$ provided answer positive.
Anything that rounds to 114 for A1.

[13]

8.	(a)	$E(X) = 3$;	B1	
		$\text{Var}(X) = \dfrac{25-1}{12} = 2$ **AG**		
		$\text{Var}(X) = 1^2 \times \frac{1}{5} + 2^2 \times \frac{1}{5} + 3^2 \times \frac{1}{5} ..... - 3^3 = 11 - 9 = 2$ **AG**	M1 A1	3
		Accept (55 / 5) – as minimum evidence.		
	(b)	$E(3X - 2) = 3E(X) - 2 = 7$	M1 A1ft	2
	(c)	$\text{Var}(4 - 3x) = 3^2\text{Var}(X) = 18$	M1 A1	2
				[7]
9.	(a)	$p + q = 0.4$	B1	
		$2p + 4q = 1.3$ Consider with (b).	M1 A1	3
	(b)	Attempt to solve	M1	
		$p = 0.15, q = 0.25$ If both seen, award 3.	A1 A1	3
	(c)	$E(X^2) = 1^2 \times 0.10 + 2^2 \times 0.15 + ... + 5^2 \times 0.30 = 14$	M1A1ft	
		$\text{Var}(X) = 14 - 3.5^2 = 1.75$	M1A1	4
	(d)	$\text{Var}(3 - 2X) = 4\text{Var}(X) = 7.00$	M1A1ft	2
				[12]
10.	(a)	$k + 2k + 3k + 5k + 6k = 1$	M1	
		use of $\Sigma P(X = x) = 1$		
		$17k = 1$		
		$k = \dfrac{1}{17} = 0.0588$	A1	2
	(b)	$E(X) = 1 \times \dfrac{1}{17} + 2 \times \dfrac{2}{17} + \ldots + 5 \times \dfrac{6}{17} = \dfrac{64}{17}$	M1	
		use of $\Sigma xP(X = x)$ and at least 2 prob correct		
		$= 3\dfrac{13}{17}$	A1	2
		Do not ignore subsequent working		
	(c)	$E(X) = 1^2 \times \dfrac{1}{17} + 2^2 \times \dfrac{2}{17} + \ldots + 5^2 \times \dfrac{6}{17} = \left(\dfrac{266}{17} = 15.6\right)$	M1 A1	

use of $\Sigma x^2 P(X = x)$ and at least 2 prob correct

Var (X) = $\frac{266}{17} - \left(\frac{64}{17}\right)^2$ M1

use of $\Sigma x^2 P(X = x)$ –

$(E(X))^2$ = 1.4740… A1 4

awrt 1.47

(d) Var (4 – 3X) = 9 Var (X) = 9 × 1.47 = 13.23 ⇒ 13.2 M1 A1 2

or 9 × 1.4740… = 13.266 ⇒ 13.3

cao 9 Var X

[10]

11. (a) $k + 2k + 3k + 4k + 5k = 1$ M1

$15k = 1$

verification / use of $\Sigma P(X = x) = 1$

** $k = \frac{1}{15}$ ** A1 2

cso

(b) $P(X < 4) = P(1) + P(2) + P(3) = \frac{1}{15} + \frac{2}{15} + \frac{3}{15}$ M1

sum of 3 probabilities

$= \frac{2}{5}$ A1 seen (2) 2

0.4 or $\frac{6}{15}$ or $\frac{2}{5}$

(c) $E(X) = 1 \times \frac{1}{15} + 2 \times \frac{2}{15} + 3 \times \frac{3}{15} + 4 \times \frac{4}{15} + 5 \times \frac{5}{15}$ M1

use of $\Sigma x P(X = x)$

$= \frac{11}{3}$ A1 2

$\frac{55}{15}$ or $\frac{11}{3}$ or $3\frac{2}{3}$ or $3.\dot{6}$ or 3.67

	(d)	$E(3X - 4) = 3E(X) - 4 = 11 - 4$	M1	
		3 × theirs-4		
		$= 7$	A1 seen (2)	
		(OR		
		$E(3X - 4) = -1 \times \frac{1}{15} + 2 \times \frac{2}{15} + 5 \times \frac{3}{15} + 8 \times \frac{4}{15} + 11 \times \frac{5}{15}$	M1	
		$\Sigma(3x - 4)kx$		
		$= 7$	A1	2
		cao		
				[8]
12.	(a)	$0.5 + b + a = 1$	M1 A1	
		use of $\Sigma P(X = x) = 1$		
		$0.3 + 2b + 3a = 1.7$	M1 A1	
		use of $E(x) = \Sigma xP(X = x)$		
		$\therefore \underline{a = 0.4 \ \& \ b = 0.1}$	B1	5
	(b)	$P(0 < X < 1.5) = P(X = 1) = \underline{0.3}$	B1	1
	(c)	$E(2X - 3) = 2E(X) - 3$	M1	
		Use of $E(aX + b)$		
		$= 2 \times 1.7 - 3 = \underline{0.4}$	A1	2
	(d)	$Var(X) = (1^2 \times 0.3) + (2^2 \times 0.1) + (3^2 \times 0.4) - 1.7^2$	M1	
		Use of $E(x^2) - \{E(x)\}^2$		
		$= 4.3 - 2.89$	A1ft	
		$= \underline{1.41}$ (*)	A1	3
		cso		
	(e)	$Var(2X - 3) = 2^2 \ Var(X)$	M1	
		Use of Var		
		$= 4 \times 1.41 = \underline{5.64}$	A1	2
				[13]

13. (a) $P(1 < X \le 3) = P(X = 2) + P(X = 3)$ M1

$= \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$ A1 2

$\frac{2}{12}$; $\frac{1}{6}$; 0.167; $0.16\dot{6}$; $0.1\dot{6}$

(b) $F(2.6) = P(X \le 2) = 1 - P(X = 3) = 1 - \frac{1}{12} = \frac{11}{12}$ B1 1

$\frac{11}{12}$; 0.917; $0.91\dot{6}$

(or: $P(X \le 2) = \frac{1}{3} + \frac{1}{2} + \frac{1}{12} = \frac{11}{12}$)

(c) $E(X) = \left(0 \times \frac{1}{3}\right) + ... + \left(3 \times \frac{1}{12}\right) = \frac{11}{12}$

Use of $\Sigma xP(X = x)$ M1

$\frac{11}{12}$; AWRT 0.917 A1 2

(d) $E(2X - 3) = 2E(X) - 3$ M1

Use of $E(ax + b)$

$= 2 \times \frac{11}{12} - 3 = -\frac{14}{12} = -\frac{7}{6}$ A1 2

$-\frac{7}{6}$; $-1\frac{1}{6}$; AWRT -1.17

(e) $Var(X) = 1^2 \times \frac{1}{2} + ... + 3^2 \times \frac{1}{12} - \left(\frac{11}{12}\right)^2$

Use of $E(X^2) - \{E(X)\}^2$ M1

Correct substitution A1ft

$= \frac{107}{144}$

$\frac{107}{144}$; AWRT 0.743 A1 3

[10]

14. (a) P(scores 30 points) = P(hit, hit, hit,) = $0.6^3 = 0.216 = \frac{27}{125}$ 0.6^3 M1

$\frac{27}{125}$; 0.216 A1 2

(b)

x	0	10	20	30
	0.4	0.6×0.4	$0.6^2 \times 0.4$	
P ($X = x$)	0.4	0.24	0.144	(0.216)
	$\frac{4}{10}$	$\frac{6}{25}$	$\frac{18}{225}$	

x = 0, 10, 20, 30 B1

One correct

P($X = x$) M1

0.4; 0.24; 0.144 A1; A1; A1 5

(c) E(X) = (0 × 0.4) + … + (30 × 0.216) = <u>11.76</u>

$\Sigma xP(X = x)$

Their distribution M1

AWRT 11.8 A1

$E(X^2) = (10^2 \times 0.24) + \ldots + (30^2 \times 0.216) = 276$ B1

Std Dev = $\sqrt{276 - 11.76^2}$ = 11.7346… $\sqrt{E(X^2) - (E(X))^2}$ M1

3 s.f. 11.7 A1 5

(d) P (Linda scores more in round 2 than in round 1)

= P(X_1 = 0 & X_2 = 10, 20, 30) $X_2 > X_1$ M1

+ P(X_1 = 10 & X_2 = 20, 30)

Can be implied

All possible A1

+P(X_1 = 20 & X_2 = 30) A1

= 0.4 × (0.24 + 0.144 + 0.216) = 0.24 A1 ft

+ 0.24 × (0.144 + 0.216) = 0.0864 A1

+ (0.144 × 0.216) = 0.031104 A1 ft

<u>= 0.357504</u> A1 6

AWRT 0.358

[18]

15. (a) $E(X) = \Sigma x \times P(X = x)$ $= \frac{1}{n} + \frac{2}{n} + \ldots\ldots + \frac{n}{n}$ *Use of E(X)* M1

Accept Verify

i.e: - $= \frac{1}{n}\{1 + 2\ldots + n\}$

$\frac{1}{9} + \frac{2}{9} + \ldots + \frac{9}{9} = \frac{45}{9} = 5$

or $\frac{n+1}{2} = 5$ $= \frac{1}{n} \cdot \frac{1}{2} n(n+1) = \frac{n+1}{2}$ M1

Use of $\frac{1}{2}n(n+1)$

$\therefore \frac{n+1}{2} = 5 \Rightarrow \underline{n = 9}$ A1 3

Must state n = 9 for final A1
c.s.o.

(b) $P(X < T) = \frac{1}{9} \times 6 = \frac{2}{3}$ M1 A1 2

$P(X \le 6)$
Use of $E(X^2)$

(c) $\text{Var}(X) = E(X^2) - \{E(X)\}^2$ A1

$\frac{95}{3} : \frac{285}{9} : 31\frac{2}{3}$

$= \frac{1^2}{9} + \frac{2^2}{9} + \ldots + \frac{9^2}{} - 5^2$ M1

Use of Var(X)

$= \frac{1}{9} \times \frac{1}{6} \times 9 \times 10 \times 19 - 5^2$ M1

$= \underline{\frac{20}{3}}$ A1 4

$6\frac{2}{3}$; $6.6\dot{6}$; 6.67; $\frac{20}{30}$

OR

$\text{Var}(X) = \frac{n^2 - 1}{12} = \frac{80}{12} = \frac{20}{3}$ M2 A1 A1

[9]

16. (a)	$k(16 - 9) + k(25 - 9) + k(36 - 9) = 1$	M1 A1	
	$\therefore\ 7k + 16k + 27k = 1 \Rightarrow k = \frac{1}{50}$	A1	3

x	4	5	6
$\text{P}(X = x)$	$\frac{7}{50}$	$\frac{16}{50}$	$\frac{27}{50}$

(b)	$\text{E}(X) = (4 \times \frac{7}{50}) + (5 \times \frac{16}{50}) + (6 \times \frac{27}{50}) = \frac{270}{50} = 5.4$		
	$\text{E}(X^2) = (4^2 \times \frac{7}{50}) + (5^2 \times \frac{16}{50}) + (6^2 \times \frac{27}{50}) = \frac{1484}{50} = 29.68$		
	$\therefore\ \text{Var}(X) = 29.68 - 5.4^2 = \dfrac{13}{25} = 0.52$		
	$\Sigma xP(X = x)$	M1	
	$\frac{27}{5}$ or 5.4	A1	
	$\Sigma x^2P(X = x)$	M1	
	29.68	A1	
	Use of $E(X^2) - \{E(x)\}^2$	M1	
	0.52	A1	6
(c)	$\text{Var}(2X - 3) = 2^2\ \text{Var}(X)$		
	$= 4 \times 0.52 = \underline{2.08}$		
	Use of $Var(x) = a^2\ Var(x)$	M1	
	+ve variance	A1 ft	2

[11]

17. (a)	Discrete uniform	B1	1
(b)	$\text{P}(X = x) = \frac{1}{6},\ x = 1, 2, \ldots, 6$		
	$\therefore\ \text{E}(X) = \Sigma x\,\text{P}(X = x) = \frac{1}{6} + \frac{2}{6} + \ldots + \frac{6}{6} = \frac{21}{6} = 3.5$	B1	
	or $\text{E}(X) = \dfrac{k+1}{2} = \dfrac{7}{2} = 3.5$		
	$\text{Var}(X) = \Sigma x^2\,\text{P}(X = x) - \{\text{E}(X)\}^2$	M1	
	$= \frac{1}{6} + \frac{4}{6} + \ldots + \frac{36}{6} - (\frac{21}{6})^2$		
	$= \frac{105}{36} = \frac{35}{12} = 2\frac{11}{12} = 2.91\dot{6}$	A1	3
	$\dfrac{35}{12},\ 2\dfrac{11}{12},\ 2.92$		
	Or $\text{Var}(X) = \dfrac{k^2 - 1}{12} = \dfrac{36 - 1}{12} = \dfrac{35}{12}$ etc		
(c)	$\text{P(three 6s)} = (\frac{1}{6})^3 = \frac{1}{216}$	M1 A1	2
(d)	$16 \Rightarrow (6, 5, 5); (5, 6, 5); (5, 5, 6)$	B1 B1	
	$(6, 6, 4); (6, 4, 6); (4, 6, 6)$	B1 B1	4

	(e)	P(16) = $\frac{6}{216}$ = $\frac{1}{36}$	M1 A1	2
				[12]
18.	(a)	$2k + k + 0 + k = 1$	M1	
		$\therefore 4k = 1 \Rightarrow k = 0.25$ **(*)**	A1	2
	(b)			

x	0	1	2	3
$P(X = x)$	0.5	0.25	0	0.25
$xP(X = x)$	0	0.25	0	0.75
$x^2P(X = x)$	0	0.25	0	2.75

		$E(X) = \Sigma xP(X = x) = 0 + 0.25 + 0 + 0.75 = 1$	M1 A1	
		$E(X^2) = 0 + 0.25 + 0 + 2.25 = 2.5$ **(*)**	M1 A1	4
	(c)	$\text{Var}(3X - 2) = 3^2\ \text{Var}(X)$	M1	
		$= 9(2.5 - 1^2) = 13.5$	M1 A1	3
	(d)	$P(X_1 + X_2) = P(X_1 = 3 \cap X_2 = 2) + P(X_1 = 2 \cap X_2 = 3) = 0 + 0 = 0$	B1	1
	(e)	Let $Y = X_1 + X_2$ y 0 1 2 3 4 5 6	B1	
		$P(Y = y)$ 0.25 0.25 0.0625 0.25 0.125 (0) 0.0625	B2	3
	(f)	$P(1.3 \le X_1 + X_2 \le 3.2) = P(X_1 + X_2 = 2) + P(X_1 + X_2 = 3)$	M1	
		$= 0.0625 + 0.25 = 0.3125$	A1 ft, A1 ft	3
				[16]
19.	(a)	P(correct at third attempt) = 0.4 × 0.4 × 0.6	M1	
		= 0.096	A1	2
	(b)			

a	1	2	3	4
$P(A = a)$	0.6	0.24	0.096	0.064

		a = 1, 2, 3, 4	B1	
		All P(A = a) correct	B1	2
	(c)	P(correct number) = $1 - (0.4)^4$	M1	
		= 0.9744		
		(accept awrt 0.974)	A12	

(d)	$E(A) = \Sigma\, a\, P(A = a) = (1 \times 0.6) + \ldots + (4 \times 0.064)$	M1	
	$= 1.624$		
	(accept awrt 1.62)	A1	
	$E(A^2) = \Sigma\, a^2\, P(A = a) = (1^2 \times 0.6) + \ldots + (4^2 \times 0.064)$	M1	
	$= 3.448$	A1	
	$\therefore$ Var $(A) = 3.448 - (1.624)^2$	M1	
	$= 0.810624$		
	(accept awrt 0.811)	A1	6
	$F(1 + E(A)) = P(A \le 1 + E(A))$		
	$= P(A \le 2.624)$	M1	
	$= 0.84$	A1	2

[14]

1. Finding the correct value of a in the first part of the question proved to be relatively straightforward for most candidates. Few errors were seen although some candidates provided very little in the way of working out and did not always make it explicit that they were using the fact that the sum of the probabilities equals one. Similarly, most candidates were able to obtain the correct value of E(X), though not many deduced this fact by recognising the symmetry of the distribution.

The majority opted to use the formula to calculate E(X), which resulted in processing errors in some cases. Common errors seen in calculating Var (X) included forgetting to subtract $[\mathrm{E}(X)]^2$ from $\mathrm{E}(X^2)$ or calculating $\mathrm{E}(X^2) - \mathrm{E}(X)$, although on the whole the correct formula was successfully applied.

Most candidates were able to correctly apply $\mathrm{Var}\,(aX + b) = a^2\,\mathrm{Var}\,(X)$ to deduce $\mathrm{Var}\,(Y) = 4\,\mathrm{Var}\,(X)$, although $\mathrm{Var}(Y) = 6 - 2\mathrm{Var}(X)$ was a typical error. Quite a number of candidates attempted to calculate $\mathrm{E}(Y^2) - [\mathrm{E}(Y)]^2$ with varying degrees of success. Occasionally, candidates divided their results in part (b), part (c) and part (d) by 5.

The final part of the question proved to be the most challenging of all and was often either completely omitted or poorly attempted with little or no success. Only a minority of candidates knew they would need to equate $6 - 2X$ to X in order to obtain the corresponding values of X and of those who did, only a small number scored full marks, as candidates were generally unable to identify the correct values of X.

2. Despite the compact nature of the probability function many candidates gave clear and fully correct solutions to this question. Part (a) was a "Show that" and candidates needed to make sure that they clearly used $\sum \mathrm{p}(x) = 1$ to form a suitable equation in k. Part (b) was often answered poorly as a number could not interpret $\mathrm{P}(X \geq 2)$ correctly and gave the answer of $\frac{5}{14}$ (from $\mathrm{P}(X \geq 2)$). Most could answer part (c) and many part (d) too but the usual errors arose here. Some forgot to subtract $(\mathrm{E}(X))^2$ and there were a number of incorrect formulae for $\mathrm{Var}(1 - X)$ seen such as: $-\mathrm{Var}(X)$ $1 - \mathrm{Var}(X)$, $[\mathrm{Var}(X)]^2$, and $(-1)^2\,\mathrm{E}(X)$.

3. This entire question was usually very well done if part (b) was correct. Some candidates did not identify that the sum of probabilities should equal one and had problems trying to find the values of a and b resorting to guessing. Even if candidates could form two correct equations, some lacked the ability to solve these relatively simple equations. A number of candidates who had no success with part (b) gave up at this point but others managed to get part (d) and part (f) correct using the values given in the question. In part (e) many knew that they had to take 1.6^2 from their figure, and not having got the figures for a and b correct in part (b) they adjusted their number to come to 4.2, so that $4.2 - 1.6^2$ came to 1.64. On occasion it was difficult to distinguish between Σ and E in the candidate's handwriting.

4. Part (a) was answered well although a small minority of candidates insisted on dividing by n (where n was usually 4). Part (b), on the other hand, caused great confusion. Some interpreted F(1.5) as E(1.5X), others interpolated between P(X = 1) and P(X = 2) and a few thought that F(1.5) was zero since X has a discrete distribution. Although the majority of candidates gained full marks in part (c) the use of notation was often poor. Statements such as Var(X) = 2 = 2 – 1 = 1 were rife and some wrote Var(X) or $\sum X^2$ when they meant E(X^2). Many candidates can now deal with the algebra of Var(X) but there were the usual errors such as 5Var(X) or 25Var(X) or –3Var(X) and the common -3^2 Var(X) which was condoned if the correct answer followed.

Part (e) was not answered well and some candidates did not attempt it. Those who did appreciate what was required often missed one or more of the possible cases or incorrectly repeated a case such as (2, 2). There were many fully correct responses though often aided by a simple table to identify the 6 cases required.

5. This proved to be a good question allowing the better students to show their understanding of the topic. It was generally done well with the majority of students aware of what they were trying to achieve. Of those who did less well, many failed to realise the significance of the 0.55 and others only came up with one equation in part (a) and thought that q was therefore equal to 0. A very large number of candidates incorrectly squared –1 in part (b) affecting their calculation of variance. Part (c) was generally well answered.

6. This question was an excellent example of why students should revise the syllabus and not just from past papers. Only a minority of candidates tackled this question effectively; some candidates seemed to have no idea at all as to how to tackle the question. Those who gave correct solutions often made many incorrect attempts in their working. The vast majority showed an understanding of discrete random variables but most missed or did not understand the word "cumulative" and consequently spent a lot of time manipulating quadratic expressions trying to make them into a probability distribution. The majority view was that F(1) + F(2) + F(3) = 1 which led to a lot of incorrect calculations.

7. A sizeable minority of candidates did not attempt this question, but, when attempted, this question was well done with many candidates picking up most or all the marks. Rather surprisingly in part (a) the equation missing was $p + q = 0.45$ and a few candidates divided one side of their second equation by 5. In part (b) nearly all of those who had two correct equations for part (a) were able to solve them simultaneously. In part (c) a substantial number of candidates were unable to make a successful attempt at this part of the question with many omitting it entirely. There were a large number of accurate solutions to part (d) with most of those making an error failing to use the given values. A number of candidates reworked E(X^2) for part (e) even though it was given. There were some mistakes in part (f) but most candidates used16 correctly, but some multiplied E(X) instead of Var(X).

8. This often scored full marks. For the variance in part (a) there were a few occasions where the working shown made it clear that the candidate would have forgotten to subtract $(E(X))^2$ if the value of the variance had not been given in the question. As is usually the case, some candidates are not aware of the need for full working when a "show that" question is asked. In part (c) some were using 4^2 rather than 3^2.

9. This proved to be a well balanced question giving strong candidates a chance to score well, but sufficient opportunities for weaker candidates to gain some marks. Some candidates appeared to guess the answers to p and q and then were able to carry these through into part (c). A significant number of candidates forgot that the total of the probabilities should be 1 and tried to 'solve' one equation while some candidates missed out the question completely. Some worked out the expectation although it was given at the start of the question.

10. Generally this question was tackled well and there was much evidence of effortless solutions.

(a) Most candidates realised that the probabilities must sum to 1 and managed to use the split definition of the probability function well. Occasionally candidates failed to use both parts of the function and focused on just kx to get a value of a sixth, or more often through incorrect calculation and ignoring the (k+1) gained a value of k = 1/15.

(b) Usually well answered, but some candidates failed to recognise the demand for an exact value and proceeded to write down an approximate equivalent decimal.

(c) This part of the question was well answered. Although a minority of candidates failed to give their answer to 1 decimal place.

11. A well answered question with many candidates scoring full marks. Some weaker candidates had difficulty in interpreting the probability function and producing a convincing argument in part (a) proved demanding for some.

12. This was a standard type of question for this paper and many of the candidates were able to answer it correctly. Some candidates wrote down one equation in part (a) and then found a and b by trial and improvement losing two marks for doing so. Some candidates did not show sufficient working in part (d) and the usual error of finding 2Var(X) instead of 4Var(X) occurred regularly in part (d).

13. A well answered question in general. However, some candidates still did not know how to deal with F(2.6) and the other common errors were to carry the wrong expected value into the final part of the question or to ignore altogether the need to square the expected value when finding the variance.

14. The first two marks were scored by many of the candidates, but in too many cases very few of the remaining marks were gained. Many candidates could not establish the values of X as 0, 10, 20 and 30 and they were unable to calculate corresponding probabilities. The methods for finding the mean and the standard deviation were usually known and they were often correctly applied to the distribution produced by the candidates. Too many candidates forgot to take the square root to find the standard deviation. Having struggled with part (b) candidates then could not interpret the demand in part (d).

15. Showing than $n = 9$ for the given discrete uniform distribution produced a variety of solutions but the mark scheme was sufficiently flexible to accommodate most of them. There were, however, too many incomplete solutions that resulted in marks being lost. Most candidates knew how to handle the rest of the question and many of them gained all the available marks in parts (b) and (c).

16. This question proved to be a good source of marks for many candidates and it was pleasing to see many candidates gaining full marks. Other candidates did not produce convincing proofs for the value of k – they had to state that $\mathrm{P}(X = x) = 1$. In part (b) there were some arithmetic slips and part (c) produced the usual crop or errors associated with $\mathrm{Var}(X)$.

17. To gain the mark in part (a) 'discrete uniform' was needed not just one of the words. It was pleasing to see that many candidates either knew the formulae for the mean and the variance in part (b) or could work them out from first principles. Part (d) proved to be a discriminator since many candidates could not give all 6 combinations that added to 16. It was, however, surprising how many did score full marks for parts (c), (d) and (e) having gained no marks for the first two parts of the question.

18. Many candidates could handle parts (a), (b) and (c) sufficiently well to gain most of the marks. The use of two independent observations from ONE random variable caused problems for many candidates. They did not realise that 5 could be made up from 2 and 3 or 3 and 2 and that since $\mathrm{P}(X = 2) = 0$ then $\mathrm{P}(X + X) = 0$. If they did not appreciate the hint given in part (d) then they were unable to continue with the question and many candidates abandoned the question at this stage. Parts (d) and (e) were very good discriminators.

19. No Report available for this question.